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Quantum scale-invariant models as effective field theories

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Abstract: We address the question of whether the quantum scale-invariant theories introduced in [1] are renormalizable or play the role of effective field theories that are valid below the Planck scale M_P. We show that starting from two-loop level the renormalization procedure requires introduction of counter-terms with structures different from those in the initial Lagrangian, making these theories non-renormalizable and therefore non-predictive above M_P. Despite non-renormalizability, the attractive features of these theories, associated with the stability of the Higgs mass agains radiative corrections and the smallness of the cosmological constant, remain intact.

Ref. [1] introduced a class of models with scale-invariance at the quantum level. Quantum scale invariance (QSI) would forbid all mass parameters in the Lagrangian, including the Higgs mass and cosmological constant, while the spontaneous breaking of QSI introduces all mass scales, including the gravitational constant. Some intriguing phenomenological consequences of these theories were discussed in [2]. They are related to the stability of the Higgs mass agains radiative corrections, to the cosmological inflation, and existence of dark energy. The key point of phenomenological considerations of [2, 1] is related to the existence of exact symmetry and its spontaneous breakdown. Though the consequences of spontaneously broken QSI theories remain valid independently of the character of these models (renormalizable theories versus effective field theories, valid up to the Planck scale $M_P \sim 10^{19}$ GeV), it is interesting to clarify whether these models are renormalizable or can only play the role of effective field theories.

A simple toy scalar field model considered in [1] is given by the action

$$\begin{aligned} S &= \int d^4x \, \mathcal{L}, \\ \mathcal{L} &= \frac{1}{2}\left[(\partial_\mu \chi)^2 + (\partial_\mu h)^2\right] - V(h,\chi), \\ V(h,\chi) &= \lambda\left(h^2 - \zeta^2\chi^2\right)^2 . \end{aligned} \tag{1}$$

It is scale-invariant at the classical level. The potential is chosen in such a way that the scale invariance is spontaneously broken along the flat directions $h = \pm\zeta\chi$. The fields h and χ can be thought of as the Higgs field and the dilaton, correspondingly. Phenomenological considerations require that one should take $\zeta \sim v/M_{\mathrm{P}} \sim 10^{-17} \lll 1$, where $v \sim 100$ GeV is the electroweak scale.

To get a scale invariant theory on the quantum level one can replace the parameter μ of dimensional regularization (we take the dimensionality of space-time equal to $n = 4 - 2\epsilon$) by an appropriate combination of dynamical fields χ and h, and remove divergences by droping the contributions that are singular in the limit $\epsilon \to 0$ [1]. This procedure was suggested already in [3], see also [4]. A choice, motivated by cosmological considerations [2], reads

$$\mu^{2\epsilon} \to \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} , \tag{2}$$

where $\omega^2 \equiv \left(\xi_\chi \chi^2 + \xi_h h^2\right)$ and $\xi_\chi,\ \xi_h$ are the couplings of the scalar fields to the gravitational Ricci scalar. The low energy theory contains a massive Higgs field and the massless dilaton.

In this short note we argue that these types of models are non-renormalizable and thus require an ultraviolet (UV) completion, at least if the standard way of renormalization is used.

To demonstrate that this is indeed the case it is sufficient to consider the simplest massless scalar model with a quartic self-interaction. It can be seen that the non-renormalizability argument holds in the more general case. We use the notations of Sec. 2 of [1] and, for convenience, start from the dimensionally regularized bare action in the form

$$S = \int d^n x \left[(\partial_\mu \varphi_B)^2 - \mu^{2\varepsilon}\lambda_B \varphi_B^4\right] . \tag{3}$$

First reprepesent the action in terms of finite fields:

$$\varphi_B\,(x) = \mu^{-\varepsilon} Z_\varphi^{\frac{1}{2}} \varphi(x)\ , \tag{4}$$

where the standard field renormalization factor is introduced. The action becomes

$$S = \int d^n x \mu^{-2\varepsilon} \left[Z_\varphi \left(\partial_\mu \varphi(x)\right)^2 - \lambda_B Z_\varphi^2 \varphi^4(x) \right]\ . \tag{5}$$

With a proper choice of the Z's and λ_B, the theory becomes finite order-by-order in powers of the coupling λ. The simplest variant of the construction of Sec. 2 of [1] is to replace

$$\mu^2 = \mu_0^2 + \xi_B \varphi^2\,(x)\ . \tag{6}$$

Within perturbation theory, perform the expansion around μ_0^2:

$$S = \int d^D x \mu_0^{-2\varepsilon} \left[Z_\varphi \left(\partial_\mu \varphi(x)\right)^2 - \lambda_B Z_\varphi^2 \varphi^4(x) - \varepsilon \mu_0^{-2} \xi_B Z_\varphi \left(\partial_\mu \varphi(x)\right)^2 \varphi^2\,(x) + O\left(\varphi^6\right) \right]\ . \tag{7}$$

One notices the presence of an abnomal vertex (the one with two derivatives and the factor ε in the coupling) along with the normal one (without derivatives, and with the usual coupling λ).

Since each UV-divergent loop in a dimensionally regulated perturbative integral (diagram) gives one factor ε^{-1}, whereas each abnormal vertex adds the factor ε, potentially interesting integrals must have the number of loops larger by one or more than the number of abnormal vertices. In a normal scalar model (no abnormal vertices), the 4-leg one-particle-irreducible diagrams diverge logarithmically. Each abnormal vertex adds a second power of momenta to the numerator of the integrand of the diagram, so that the index of UV divergence is increased by two. Since the action already contains a vertex with two derivatives, such divergences can be absorbed into the factor ξ_B. Therefore, problematic divergences can emerge only in diagrams with two or more abnormal vertices.

We conclude that although there are no problems with UV renormalization of the model (3) at the one- and two-loop level, there are non-renormalizable UV divergences starting from three loop level upwards. One can see that the same conclusion holds for more complex models (non-zero masses, more fields, shifts of the fields by c-number constants). A similar argument in the case of fermions Yukawa-coupled to scalars, establishes that the nonrenormalizable contributions begin to occur at the two-loop level; the same is true for scalar theories with massive fields. If one introduces gauge fields into the model, then the one-loop renormalizability argument of [1] remains correct, whereas starting from two-loop level, non-renormalizable UV divergences start to occur via the outlined mechanism.

Let us turn now back to the Lagrangian (1). In the Higgs sector (no dilatons in external legs, which are suppressed by the small value of ζ) the contribution which is leading in momenta comes from the three-loop graph shown in Fig. 1. It contains two normal vertices λh^4 and one abnormal vertex, coming from the expansion of μ with respect to h in (2).

It is therefore necessary to introduce a counterterm with the structure

$$\left(\frac{1}{16\pi^2}\right)^3 \frac{1}{\varepsilon^2} \left(\frac{\xi_h}{\xi_\chi \chi^2}\right)^2 \left[\Box h^2\right]^2\ . \tag{8}$$

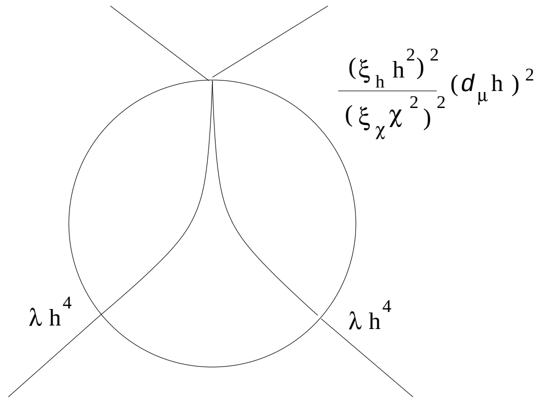

Figure 1: A three loop contribution to higher-dimensional operators that is leading in momenta.

This leads to an estimate of the energy domain of validity of the QSI effective field theory associated with the action (1):

$$E^2 \lesssim \frac{(16\pi^2)^2}{\lambda}\frac{\xi_\chi \chi_0^2}{\xi_h}, \tag{9}$$

which is of the order of the Planck scale.

The counter-term Lagrangian L_{ct}, removing ultraviolet divergencies, can be made local in the following sense. The perturbative construction of scale-invariant theories implies the separation of the quantum fields into a c-number part related to the background χ_0, h_0, which breaks the QSI, and perturbations, $\chi = \chi_0 + \delta\chi, \quad h = h_0 + \delta h$. If L_{ct} satisfies

$$\left[\frac{\partial}{\partial \chi_0} - \frac{\delta}{\delta(\delta\chi)}\right] L_{ct} = 0 \ , \qquad \left[\frac{\partial}{\partial h_0} - \frac{\delta}{\delta(\delta h)}\right] L_{ct} = 0 \ , \tag{10}$$

then it will be a local function of the fields χ and h (i.e. depend only on the combinations $\chi_0 + \delta\chi$ and $h = h_0 + \delta h$. The fact that the regularized ($n \neq 4$) but non-renormalized Green's functions of the theory (1) with prescription (2) satisfy the equations (10) ensures that this is indeed the case.

To conclude, the QSI theories with spontaneous breaking of scale invariance are predictive at energies $E \ll M_P$ but require an ultraviolet completion at Planck energies. It is intriguing that in order to make these theories compartible with the tests of General Relativity, the gravity should be introdiced in a scale-invariant way [5, 6, 2]. This leads to derivative coupling of the dilaton to matter fields, evading the fifth-force bounds on the massless scalars.

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